NCERT Solutions for Class 11 Maths Chapter 2

Relations and Functions Class 11

Chapter 2 Relations and Functions Exercise 2.1, 2.2, 2.3, miscellaneous Solutions

Exercise 2.1 : Solutions of Questions on Page Number : **33**

Q1 :

If $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$**, find the values of *x*and *y*.**

Answer :

It is given that $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$.

$$\begin{aligned}&\frac{x}{3}+1=\frac{5}{3}\\&\Rightarrow \frac{x}{3}=\frac{5}{3}-1\\&\Rightarrow \frac{x}{3}=\frac{2}{3}\\&\Rightarrow x=2\end{aligned}
\qquad
\begin{aligned}&y-\frac{2}{3}=\frac{1}{3}\\&\Rightarrow y=\frac{1}{3}+\frac{2}{3}\\&\Rightarrow y=1\end{aligned}$$

$\therefore$ *x*= 2 and *y*= 1

Q2 :

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A x B)?

Answer :

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

⇒ Number of elements in set B = 3

Number of elements in (A x B)

= (Number of elements in A) x (Number of elements in B)

= 3 x 3 = 9

Thus, the number of elements in (A x B) is 9.

Q3 :

If G = {7, 8} and H = {5, 4, 2}, find G x H and H x G.

Answer :

G = {7, 8} and H = {5, 4, 2}

We know that the Cartesian product P x Q of two non-empty sets P and Q is defined as

P x Q = {(p, q): p∈P, q ∈Q}

∴G x H = {(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)}

H x G = {(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)}

Q4 :

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {*m*, *n*} and Q = {*n*, *m*}, then P x Q = {(*m*, *n*), (*n*, *m*)}.

(ii) If A and B are non-empty sets, then A x B is a non-empty set of ordered pairs (*x*, *y*) such that *x* ∈ A and *y* ∈ B.

(iii) If A = {1, 2}, B = {3, 4}, then A x (B ∩ Φ) = Φ.

Answer :

(i) False

If P = {*m*, *n*} and Q = {*n*, *m*}, then

P x Q = {(*m*, *m*), (*m*, *n*), (*n*, *m*), (*n*, *n*)}

(ii) True

(iii) True

Q5 :

If A = {-1, 1}, find A x A x A.

Answer :

It is known that for any non-empty set A, A x A x A is defined as

A x A x A = {(*a*, *b*, *c*): *a*, *b*, *c* ∈ A}

It is given that A = {-1, 1}

∴ A x A x A = {(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),

(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)}

Q6 :

If A x B = {(*a*, *x*), (*a*, *y*), (*b*, *x*), (*b*, *y*)}. Find A and B.

Answer :

It is given that A x B = {(*a*, *x*), (*a*, *y*), (*b*, *x*), (*b*, *y*)}

We know that the Cartesian product of two non-empty sets P and Q is defined as P x Q = {(*p*, *q*): *p* ∈P, *q* ∈Q}

∴ A is the set of all first elements and B is the set of all second elements.

Thus, A = {*a*, *b*} and B = {*x*, *y*}

Q7 :

Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that

(i) A x (B ∩C) = (A x B) ∩(A x C)

(ii) A x C is a subset of B x D

Answer :

(i) To verify: A x (B ∩C) = (A x B) ∩(A x C)

We have B ∩C = {1, 2, 3, 4} ∩{5, 6} = Φ

∴L.H.S. = A x (B ∩C) = A x Φ = Φ

A x B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

A x C = {(1, 5), (1, 6), (2, 5), (2, 6)}

∴ R.H.S. = (A x B) ∩(A x C) = Φ

∴L.H.S. = R.H.S

Hence, A x (B ∩C) = (A x B) ∩(A x C)

(ii) To verify: A x C is a subset of B x D

A x C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B x D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

We can observe that all the elements of set A x C are the elements of set B x D.

Therefore, A x C is a subset of B x D.

Q8 :

Let A = {1, 2} and B = {3, 4}. Write A x B. How many subsets will A x B have? List them.

Answer :

A = {1, 2} and B = {3, 4}

∴A x B = {(1, 3), (1, 4), (2, 3), (2, 4)}

⇒ *n*(A x B) = 4

We know that if C is a set with *n*(C) = *m*, then *n*[P(C)] = 2^m.

Therefore, the set A x B has 2^4= 16 subsets. These are

Φ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)},

{(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)},

{(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4)}, {(1, 3), (2, 3), (2, 4)},

{(1, 4), (2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3), (2, 4)}

Q9 :

Let A and B be two sets such that *n*(A) = 3 and *n* (B) = 2. If (*x*, 1), (*y*, 2), (*z*, 1) are in A x B, find A and B, wherex, *y* and *z* are distinct elements.

Answer :

It is given that *n*(A) = 3 and *n*(B) = 2; and (*x*, 1), (*y*, 2), (*z*, 1) are in A x B.

We know that A = Set of first elements of the ordered pair elements of A x B

B = Set of second elements of the ordered pair elements of A x B.

∴ *x*, *y*, and *z*are the elements of A; and 1 and 2 are the elements of B.

Since *n*(A) = 3 and *n*(B) = 2, it is clear that A = {*x*, *y*, *z*} and B = {1, 2}.

Q10 :

The Cartesian product A x A has 9 elements among which are found (-1, 0) and (0, 1). Find the set A and the remaining elements of A x A.

Answer :

We know that if *n*(A) = *p* and *n*(B) = *q,* then *n*(A x B) = *pq*.

∴ *n*(A x A) = *n*(A) x *n*(A)

It is given that *n*(A x A) = 9

∴ *n*(A) x *n*(A) = 9

⇒ *n*(A) = 3

The ordered pairs (-1, 0) and (0, 1) are two of the nine elements of A x A.

We know that A x A = {(*a, a*): *a* ∈A}. Therefore, -1, 0, and 1 are elements of A.

Since *n*(A) = 3, it is clear that A = {-1, 0, 1}.

The remaining elements of set A x A are (-1, -1), (-1, 1), (0, -1), (0, 0),

(1, -1), (1, 0), and (1, 1)

Exercise 2.2 : Solutions of Questions on Page Number : **35**

Q1 :

Let A = {1, 2, 3, ... , 14}. Define a relation R from A to A by R = {(*x*, *y*): 3*x* - *y* = 0, where *x*, *y* ∈ A}. Write down its domain, codomain and range.

Answer :

The relation R from A to A is given as

R = {(*x*, *y*): 3*x* - *y* = 0, where *x*, *y* ∈ A}

i.e., R = {(*x*, *y*): 3*x* = *y*, where *x*, *y* ∈ A}

∴R = {(1, 3), (2, 6), (3, 9), (4, 12)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴Domain of R = {1, 2, 3, 4}

The whole set A is the codomainof the relation R.

∴Codomain of R = A = {1, 2, 3, ..., 14}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴Range of R = {3, 6, 9, 12}

Q2 :

Define a relation R on the set Nof natural numbers by R = {(*x*, *y*): *y*= *x*+ 5, *x*is a natural number less than 4; *x*,*y* ∈ N}. Depict this relationship using roster form. Write down the domain and the range.

Answer :

R = {(*x*, *y*): *y*= *x*+ 5, *x*is a natural number less than 4, *x*, *y* ∈ **N**}

The natural numbers less than 4 are 1, 2, and 3.

∴R = {(1, 6), (2, 7), (3, 8)}

The domain of R is the set of all first elements of the ordered pairs in the relation.

∴ Domain of R = {1, 2, 3}

The range of R is the set of all second elements of the ordered pairs in the relation.

∴ Range of R = {6, 7, 8}

Q3 :

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(*x*, *y*): the difference between *x*and *y*is odd; *x* ∈A, *y* ∈B}. Write R in roster form.

Answer :

A = {1, 2, 3, 5} and B = {4, 6, 9}

R = {(*x*, *y*): the difference between *x*and *y*is odd; *x* ∈A, *y* ∈B}

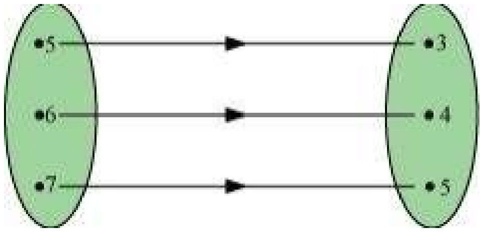
∴R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Q4 :

The given figure shows a relationship between the sets P and Q. write this relation

(i) in set-builder form (ii) in roster form.

What is its domain and range?



Answer :

According to the given figure, P = {5, 6, 7}, Q = {3, 4, 5}

(i) R = {(*x, y*): *y* = *x*- 2; *x* ∈P} or R = {(*x, y*): *y* = *x*- 2 for *x*= 5, 6, 7}

(ii) R = {(5, 3), (6, 4), (7, 5)}

Domain of R = {5, 6, 7}

Range of R = {3, 4, 5}

Q5 :

Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by

{(*a*, *b*): *a*, *b* ∈A, *b*is exactly divisible by *a*}.

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R.

Answer :

A = {1, 2, 3, 4, 6}, R = {(*a*, *b*): *a*, *b* ∈A, *b*is exactly divisible by *a*}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

Q6 :

Determine the domain and range of the relation R defined by R = {(*x*, *x*+ 5): *x* ∈{0, 1, 2, 3, 4, 5}}.

Answer :

R = {(*x*, *x*+ 5): *x* ∈{0, 1, 2, 3, 4, 5}}

∴ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

∴Domain of R = {0, 1, 2, 3, 4, 5}

Range of R = {5, 6, 7, 8, 9, 10}

Q7 :

Write the relation R = {(x, x^3): *x* is a prime number less than 10} in roster form.

Answer :

R = {(x, x^3): *x* is a prime number less than 10}

The prime numbers less than 10 are 2, 3, 5, and 7.

∴R = {(2, 8), (3, 27), (5, 125), (7, 343)}

Q8 :

Let A = {*x*, *y*, z} and B = {1, 2}. Find the number of relations from A to B.

Answer :

It is given that A = {*x*, *y*, z} and B = {1, 2}.

∴ A x B = {(*x*, 1), (*x*, 2), (*y*, 1), (*y*, 2), (*z*, 1), (*z*, 2)}

Since *n*(A x B) = 6, the number of subsets of A x B is 2^6.

Therefore, the number of relations from A to B is 2^6.

Q9 :

Let R be the relation on Zdefined by R = {(*a*, *b*): *a*, *b* ∈ Z, *a* - *b*is an integer}. Find the domain and range of R.

Answer :

R = {(*a*, *b*): *a*, *b* ∈ **Z**, *a* - *b*is an integer}

It is known that the difference between any two integers is always an integer.

∴Domain of R = **Z**

Range of R = **Z**

Exercise 2.3 : Solutions of Questions on Page Number : **44**

Q1 :

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Answer :

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Q2 :

Find the domain and range of the following real function:

(i) *f*(*x*) = â€“|*x*| (ii) $f(x)=\sqrt{9-x^2}$

Answer :

(i) *f*(*x*) = â€“|*x*|, *x* ∈R

We know that |*x*| = $\begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \ge 0 \\ x, & x < 0 \end{cases}$$

Since *f*(*x*) is defined for *x* ∈ **R**, the domain of *f*is **R.**

It can be observed that the range of *f*(*x*) = â€“|*x*| is all real numbers except positive real numbers.

∴The range of *f*is (â€“ ∞, 0].

(ii) $f(x)=\sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to â€“3 and less than or equal to 3, the domain of *f*(*x*) is {*x* : â€“3 ≤ *x* ≤ 3} or [â€“3, 3].

For any value of *x*such that â€“3 ≤ *x*≤ 3, the value of *f*(*x*) will lie between 0 and 3.

∴The range of *f*(*x*) is {*x*: 0 ≤ *x* ≤ 3} or [0, 3].

Q3 :

A function *f*is defined by *f*(*x*) = 2*x*- 5. Write down the values of

(i) *f*(0), (ii) *f*(7), (iii) *f*(-3)

Answer :

The given function is *f*(*x*) = 2*x*- 5.

Therefore,

(i) *f*(0) = 2 x 0 - 5 = 0 - 5 = -5

(ii) *f*(7) = 2 x 7 - 5 = 14 - 5 = 9

(iii) *f*(-3) = 2 x (-3) - 5 = - 6 - 5 = -11

Q4 :

The function '*t*' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C)=\frac{9C}{5}+32$**.**

Find (i) *t*(0) (ii) *t*(28) (iii) *t*(â€“10) (iv) The value of C, when *t*(C) = 212

Answer :

The given function is $t(C)=\frac{9C}{5}+32$.

Therefore,

(i) $t(0)=\frac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\frac{9\times 28}{5}+32=\frac{252+160}{5}=\frac{412}{5}$

(iii) $t(-10)=\frac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that *t*(C) = 212

$$\therefore 212=\frac{9C}{5}+32$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow \frac{9C}{5}=180$$
$$\Rightarrow 9C=180\times 5$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of *t*,when *t*(C) = 212, is 100.

Q5 :

Find the range of each of the following functions.

(i) *f*(*x*) = 2 - 3*x*, *x* ∈ R, *x*> 0.

(ii) *f*(*x*) = *x*²+ 2, *x*, is a real number.

(iii) *f*(*x*) = *x*, *x*is a real number

Answer :

(i) *f*(*x*) = 2 â€“ 3*x*, *x* ∈ **R**, *x*> 0

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
$f(x)$	1.97	1.7	â€“0.7	â€“1	â€“4	â€“5.5	â€“10	â€“13	…

Thus, it can be clearly observed that the range of *f* is the set of all real numbers less than 2.

i.e., range of *f* = (â€“ ∞, 2)

Alter:

Let $x > 0$

⇒$3x > 0$

⇒ 2 â€“3*x* < 2

⇒ $f(x) < 2$

∴Range of *f* = (â€“ ∞, 2)

(ii) $f(x) = x^2 + 2$, x, is a real number

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	±0.3	±0.8	±1	±2	±3		…
$f(x)$	2	2.09	2.64	3	6	11		…..

Thus, it can be clearly observed that the range of *f* is the set of all real numbers greater than 2.

i.e., range of *f* = [2, ∞)

Alter:

Let x be any real number.

Accordingly,

$x^2 \geq 0$

⇒ $x^2 + 2 \geq 0 + 2$

⇒ $x^2 + 2 \geq 2$

⇒ $f(x) \geq 2$

∴ Range of *f* = [2, ∞)

(iii) $f(x) = x$, x is a real number

It is clear that the range of *f* is the set of all real numbers.

∴ Range of *f* = **R**

Exercise Miscellaneous : Solutions of Questions on Page Number : **46**

Q1 :

The relation *f* is defined by $f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$

The relation *g*is defined by $g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$

Show that *f* is a function and *g* is not a function.

Answer :

The relation *f*is defined as $f(x)=\begin{cases} x^2, & 0\le x\le 3 \\ 3x, & 3\le x\le 10 \end{cases}$

It is observed that for

$0 \le x < 3$, $f(x) = x^2$

$3 < x \le 10$, $f(x) = 3x$

Also, at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.

Thus, the given relation is a function.

The relation *g*is defined as $g(x)=\begin{cases} x^2, & 0\le x\le 2 \\ 3x, & 2\le x\le 10 \end{cases}$

It can be observed that for $x= 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation *g*corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function.

Q2 :

If $f(x) = x^2$, find $\dfrac{f(1.1)-f(1)}{(1.1-1)}$.

Answer :

$$f(x)=x^2$$

$$\therefore \frac{f(1.1)-f(1)}{(1.1-1)}=\frac{(1.1)^2-(1)^2}{(1.1-1)}=\frac{1.21-1}{0.1}=\frac{0.21}{0.1}=2.1$$

Q3 :

Find the domain of the function $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

Answer :

The given function is $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$.

$$f(x)=\frac{x^2+2x+1}{x^2-8x+12}=\frac{x^2+2x+1}{(x-6)(x-2)}$$

It can be seen that function *f*is defined for all real numbers except at $x= 6$ and $x= 2$.

Hence, the domain of *f*is **R** â€“ {2, 6}.

Q4 :

Find the domain and the range of the real function *f*defined by $f(x)=\sqrt{(x-1)}$**.**

Answer :

The given real function is $f(x)=\sqrt{x-1}$.

It can be seen that $\sqrt{x-1}$ is defined for (*x*â€“ 1) ≥0.

$$f(x)=\sqrt{(x-1)}$$

∞

$$\sqrt{x-1}\geq 0$$

∞

i.e., is defined for $x \geq 1$.

Therefore, the domain of *f* is the set of all real numbers greater than or equal to 1 i.e., the domain of *f* = [1,).

As $x \geq 1 \Rightarrow (x$ â€“ $1) \geq 0 \Rightarrow$

Therefore, the range of *f* is the set of all real numbers greater than or equal to 0 i.e., the range of *f* = [0,).

Q5 :

Find the domain and the range of the real function *d*efined by *f*(*x*) = |*x*- 1|.

Answer :

The given real function is $f(x) = |x - 1|$.

It is clear that $|x - 1|$ is defined for all real numbers.

∴Domain of *f* = **R**

Also, for $x \in$ **R**, $|x - 1|$ assumes all real numbers.

Hence, the range of *f* is the set of all non-negative real numbers.

Q6 :

Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$ **be a function from R into R. Determine the range of *f*.**

Answer :

$$f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in \mathbf{R}\right\}$$

$$= \left\{(0, 0), \left(\pm 0.5, \frac{1}{5}\right), \left(\pm 1, \frac{1}{2}\right), \left(\pm 1.5, \frac{9}{13}\right), \left(\pm 2, \frac{4}{5}\right), \left(3, \frac{9}{10}\right), \left(4, \frac{16}{17}\right), \ldots\right\}$$

The range of *f* is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1.

[Denominator is greater numerator]

Thus, range of *f* = [0, 1)

Q7 :

Let *f*, *g*: R Ã¢â€ ’ R be defined, respectively by *f*(*x*) = *x* + 1, *g*(*x*) = 2*x* â€“ 3. Find *f*+ *g*, *f*â€“ *g*and $\frac{f}{g}$.

Answer :

f, *g*: **R** Ã¢â€ ’ **R** is defined as *f*(*x*) = *x* + 1, *g*(*x*) = 2*x* â€“ 3

(*f*+ *g*) (*x*) = *f*(*x*) + *g*(*x*) = (*x* + 1) + (2*x* â€“ 3) = 3*x* â€“ 2

∴(*f* + *g*) (*x*) = 3*x* â€“ 2

(*f* â€“ *g*) (*x*) = *f*(*x*) â€“ *g*(*x*) = (*x* + 1) â€“ (2*x*â€“ 3) = *x*+ 1 â€“ 2*x*+ 3 = â€“ *x*+ 4

∴ (*f* â€“ *g*) (*x*) = â€“*x*+ 4

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)},\ g(x)\neq 0,\ x\in\mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3},\ 2x-3\neq 0 \text{ or } 2x\neq 3$$

$$\therefore \left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3},\ x\neq\frac{3}{2}$$

Q8 :

Let *f* = {(1, 1), (2, 3), (0, -1), (-1, -3)} be a function from Zto Zdefined by *f*(*x*) = *ax*+ *b*, for some integers *a*, *b*. Determine *a*, *b*.

Answer :

f = {(1, 1), (2, 3), (0, -1), (-1, -3)}

f(*x*) = *ax*+ *b*

(1, 1) ∈ *f*

⇒ *f*(1) = 1

⇒ *a* x 1 + *b*= 1

⇒ *a*+ *b*= 1

(0, -1) ∈ *f*

⇒ *f*(0) = -1

⇒ *a* x 0 + *b*= -1

⇒ *b*= -1

On substituting *b*= -1 in *a*+ *b*= 1, we obtain *a*+ (-1) = 1 ⇒ *a*= 1 + 1 = 2.

Thus, the respective values of *a*and *b*are 2 and -1.

Q9 :

Let R be a relation from N to N defined by R = {(*a*, *b*): *a*, *b* ∈ N and *a* = b^2}. Are the following true?

(i) (*a*, *a*) ∈ R, for all *a* ∈ N
(ii) (*a*, *b*) ∈ R, implies (*b*, *a*) ∈ R

(iii) (*a*, *b*) ∈ R, (*b*, *c*) ∈ R implies (*a*, *c*) ∈ R.

Justify your answer in each case.

Answer :

R = {(a, b): a, b ∈ **N** and $a = b^2$}

(i) It can be seen that 2 ∈ **N**;however, $2 \neq 2^2 = 4$.

Therefore, the statement "(a, a) ∈ R, for all a ∈ **N**" is not true.

(ii) It can be seen that (9, 3) ∈ **N** because 9, 3 ∈ **N** and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, (3, 9) ∠"° **N**

Therefore, the statement "(a, b) ∈ R, implies (b, a) ∈ R" is not true.

(iii) It can be seen that (16, 4) ∈ R, (4, 2) ∈ R because 16, 4, 2 ∈ **N** and $16 = 4^2$ and $4 = 2^2$.

Now, $16 \neq 2^2 = 4$; therefore, (16, 2) ∠"° **N**

Therefore, the statement "(a, b) ∈ R, (b, c) ∈ R implies (a, c) ∈ R" is not true.

Q10 :

Let A = {1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16} and *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}. Are the following true?

(i) *f*is a relation from A to B (ii) *f*is a function from A to B.

Justify your answer in each case.

Answer :

A = {1, 2, 3, 4} and B = {1, 5, 9, 11, 15, 16}

∴A x B = {(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)}

It is given that *f* = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A x B.

It is observed that *f*is a subset of A x B.

Thus, *f*is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation *f* is not a function.

Q11 :

Let *f*be the subset of Z x Zdefined by *f* = {(ab, a+ b): a, b ∈ Z}. Is *f*a function from Zto Z: justify your answer.

Answer :

The relation *f*is defined as *f* = {(ab, a+ b): a, b ∈ **Z**}

We know that a relation *f*from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since 2, 6, -2, -6 ∈ **Z**, (2 x 6, 2 + 6), (-2 x -6, -2 + (-6)) ∈ *f*

i.e., (12, 8), (12, -8) ∈ *f*

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation *f*is not a function.

Q12 :

Let A = {9, 10, 11, 12, 13} and let *f*: A → Nbe defined by *f*(*n*) = the highest prime factor of *n*. Find the range of*f*.

Answer :

A = {9, 10, 11, 12, 13}

f: A → **N**is defined as

f(*n*) = The highest prime factor of *n*

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

∴*f*(9) = The highest prime factor of 9 = 3

f(10) = The highest prime factor of 10 = 5

f(11) = The highest prime factor of 11 = 11

f(12) = The highest prime factor of 12 = 3

f(13) = The highest prime factor of 13 = 13

The range of *f*is the set of all *f*(*n*), where *n* ∈A.

∴Range of *f*= {3, 5, 11, 13}