NCERT Solutions for Class 10 Maths Unit 2

Polynomials Class 10

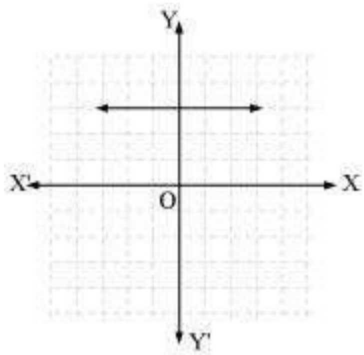
Unit 2 Polynomials Exercise 2.1, 2.2, 2.3 2.4, 2.4 Solutions

Exercise 2.1 : Solutions of Questions on Page Number : **28**

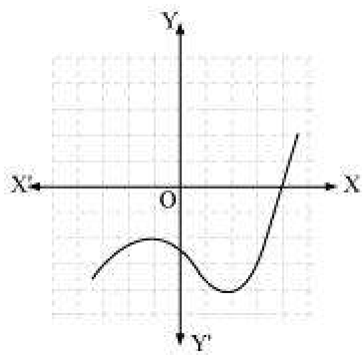
Q1 :

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of$p(x)$, in each case.

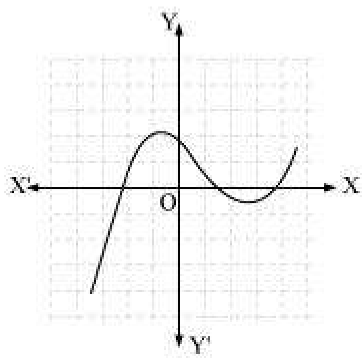
(i)



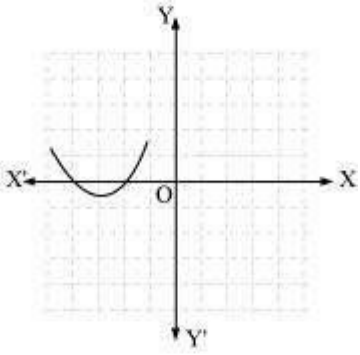
(ii)



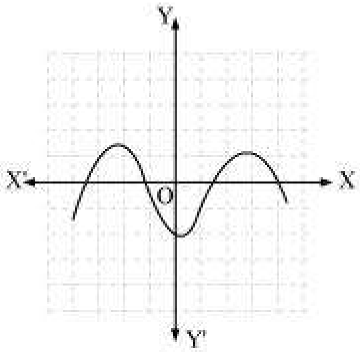
(iii)



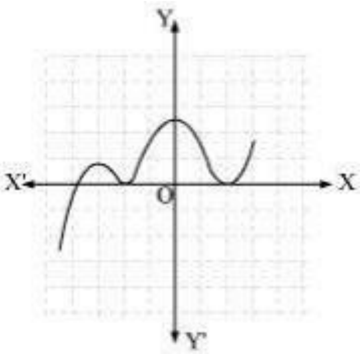
(iv)



(v)



(v)



Answer :

(i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

Q1 :

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8 \quad (\text{ii})\, 4s^2-4s+1 \quad (\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u \quad (\text{v})\, t^2-15 \quad (\text{vi})\, 3x^2-x-4$

Answer :

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when x - 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = - 2

Therefore, the zeroes of x^2-2x-8 are 4 and - 2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2$ - $4s$ + 1 is zero when $2s$ - 1 = 0, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2$ - $4s$ + 1 are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2$ - 3 - $7x$ is zero when $3x$ + 1 = 0 or $2x$ - 3 = 0, i.e., $x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2$ - 3 - $7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2+8u=4u^2+8u+0$

$=4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and - 2.

Sum of zeroes = $0+(-2)=-2=\frac{-(8)}{4}=\frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) t^2-15

$=t^2-0t-15$

$=(t-\sqrt{15})(t+\sqrt{15})$

The value of t^2 - 15 is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when

Q2 :

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Answer :

(i) $\frac{1}{4},-1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$$

$$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$$

If $a=4$, then $b=-1$, $c=-4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$$

$$\alpha\beta=\frac{1}{3}=\frac{c}{a}$$

If $a=3$, then $b=-3\sqrt{2}$, $c=1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=0=\frac{0}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=\sqrt{5}=\frac{\sqrt{5}}{1}=\frac{c}{a}$$

If $a=1$, then $b=0$, $c=\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) $1, 1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If $a=1$, then $b=-1$, $c=1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and

Exercise 2.3 2.4 : Solutions of Questions on Page Number : **36**

Q1 :

Divide the polynomial *p*(*x*) by the polynomial *g*(*x*) and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3, \qquad g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5, \qquad g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6, \qquad g(x)=2-x^2$

Answer :

$$\begin{aligned}(\text{i}) \quad & p(x)=x^3-3x^2+5x-3\\ & q(x)=x^2-2\end{aligned}$$

$$\begin{array}{r|l} & x-3 \\ \hline x^2-2 & x^3-3x^2+5x-3 \\ & x^3 \qquad\quad -2x \\ & - \qquad\quad\;\; + \\ \hline & \quad -3x^2+7x-3 \\ & \quad -3x^2 \qquad +6 \\ & \quad + \qquad\quad\;\; - \\ \hline & \qquad\qquad 7x-9 \\ \hline \end{array}$$

Quotient = x - 3

Remainder = 7x - 9

$$\begin{aligned}(\text{ii}) \quad & p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5\\ & q(x)=x^2+1-x=x^2-x+1\end{aligned}$$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\
\hline
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \quad - \quad + \\
\hline
 & 8 \\
\hline
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\hline
 & 2x^2 - 5x + 6 \\
 & 2x^2 \qquad - 4 \\
 & - \qquad + \\
\hline
 & -5x + 10 \\
\hline
\end{array}
$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Q2 :

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Answer :

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$
$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$
$$= 0$$
$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$

Therefore, $\frac{1}{2}$, 1, and - 2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain a = 2, b = 1, c = - 5, d = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$
$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2 = 0$$
$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$
$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1) = 2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Q3 :

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer :

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$t^2 - 3 = t^2 + 0.t - 3$

$$\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
\hline
t^2 + 0.t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0.t^3 - 6t^2 \\
 & - \quad - \quad + \\
\hline
 & 3t^3 + 4t^2 - 9t - 12 \\
 & 3t^3 + 0.t^2 - 9t \\
 & - \quad - \quad + \\
\hline
 & 4t^2 + 0.t - 12 \\
 & 4t^2 + 0.t - 12 \\
 & - \quad - \quad + \\
\hline
 & 0
\end{array}$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & + \quad + \quad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -x^3 \qquad + 3x + 1 \\
 & -x^3 \qquad + 3x - 1 \\
 & + \qquad - \quad + \\
\hline
 & 2 \\
\hline
\end{array}
$$

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q4 :

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer :

Let the polynomial be ax^3+bx^2+cx+d and the zeroes be α, β, and γ.

It is given that

$$\alpha+\beta+\gamma=\frac{2}{1}=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{-7}{1}=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{-14}{1}=\frac{-d}{a}$$

If a = 1, then b = - 2, c = - 7, d = 14

Hence, the polynomial is $x^3-2x^2-7x+14$.

Q5 :

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer :

$$p(x)=3x^4+6x^3-2x^2-10x-5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$\begin{array}{r|l} & 3x^2+6x+3 \\ \hline x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\ & 3x^4+0x^3-5x^2 \\ & -\quad -\quad + \\ \hline & \quad 6x^3+3x^2-10x-5 \\ & \quad 6x^3+0x^2-10x \\ & \quad -\quad -\quad + \\ \hline & \qquad 3x^2+0x-5 \\ & \qquad 3x^2+0x-5 \\ & \qquad -\quad -\quad + \\ \hline & \qquad\quad 0 \end{array}$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$
$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, - 1 and - 1.

Q6 :

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $- 2x + 4$, respectively. Find $g(x)$.

Answer :

$p(x)=x^3-3x^2+x+2$ (Dividend)

$g(x) = ?$ (Divisor)

Quotient = $(x - 2)$

Remainder = (- 2x + 4)

Dividend = Divisor × Quotient + Remainder

$$x^3-3x^2+x+2=g(x)\times(x-2)+(-2x+4)$$
$$x^3-3x^2+x+2+2x-4=g(x)(x-2)$$
$$x^3-3x^2+3x-2=g(x)(x-2)$$

$g(x)$ is the quotient when we divide (x^3-3x^2+3x-2) by $(x-2)$

$$\begin{array}{r|l} & x^2-x+1 \\ \hline x-2 & x^3-3x^2+3x-2 \\ & x^3-2x^2 \\ & -\quad + \\ \hline & -x^2+3x-2 \\ & -x^2+2x \\ & +\quad - \\ \hline & x-2 \\ & x-2 \\ & -\quad + \\ \hline & 0 \\ \hline \end{array}$$

$$\therefore g(x)=(x^2-x+1)$$

Q7 :

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x)$ = 0

Answer :

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x)$ = 0 or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.

Here, $p(x) = 6x^2+2x+2$

$g(x) = 2$

$q(x) = 3x^2+x+1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2+2x+2 = 2(3x^2+x+1)$

$= 6x^2+2x+2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of x^3+x by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3+1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4 : Solutions of Questions on Page Number : **37**

Q1 :

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a-b, a, a+b$, find *a* and *b*.

Answer :

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are *a* - *b*, *a* + *a* + *b*

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

p = 1, *q* = - 3, *r* = 1, *t* = 1

$\text{Sum of zeroes} = a - b + a + a + b$

$\frac{-q}{p} = 3a$

$\frac{-(-3)}{1} = 3a$

$3 = 3a$

$a = 1$

The zeroes are $1-b, 1, 1+b$.

$\text{Multiplication of zeroes} = 1(1-b)(1+b)$

$\frac{-t}{p} = 1 - b^2$

$\frac{-1}{1} = 1 - b^2$

$1 - b^2 = -1$

$1 + 1 = b^2$

$b = \pm\sqrt{2}$

Hence, *a* = 1 and *b* = $\sqrt{2}$ or $-\sqrt{2}$.

Q2 :

]It two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$, find other zeroes.

Answer :

Given that 2 + $\sqrt{3}$ and 2 $-\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)$ = $x^2 + 4 - 4x - 3$

= $x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4-6x^3-26x^2+138x-35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+x^2 \\
 & -\quad +\quad - \\
\hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-2x \\
 & +\quad -\quad + \\
\hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & +\quad -\quad + \\
\hline
 & 0
\end{array}
$$

Clearly, $x^4-6x^3-26x^2+138x-35$ = $\left(x^2-4x+1\right)\left(x^2-2x-35\right)$

It can be observed that $\left(x^2-2x-35\right)$ is also a factor of the given polynomial.

And $\left(x^2-2x-35\right)$ = $\left(x-7\right)\left(x+5\right)$

Therefore, the value of the polynomial is also zero when $x-7=0$ or $x+5=0$

Or x = 7 or - 5

Hence, 7 and - 5 are also zeroes of this polynomial.

Q3 :

If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be *x* + *a*, find *k* and *a*.

Answer :

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend - Remainder = Divisor × Quotient

$x^4-6x^3+16x^2-25x+10-x-a=x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by x^2-2x+k.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by x^2-2x+k

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+\ kx^2 \\
 & -\ \ +\ \ - \\
\hline
 & -4x^3+(16-k)x^2-26x \\
 & -4x^3+\ 8x^2-\ 4kx \\
 & +\ \ -\ \ + \\
\hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & -\ \ +\ \ - \\
\hline
 & (-10+2k)x+(10-a-8k+k^2)
\end{array}
$$

$$(-10+2k)x+(10-a-8k+k^2)$$

It can be observed that $(-10+2k)$ $(10-a-8k+k^2)$ will be 0.

Therefore, $(-10+2k)$ = 0 and = 0

For = 0,

2 *k* =10

And thus, *k* = 5

$(10-a-8k+k^2)$

For = 0

10 - *a* - 8 × 5 + 25 = 0

10 - *a* - 40 + 25 = 0

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$