NCERT Solutions for Class 11 Maths Chapter 12

Introduction to Three Dimensional Geometry Class 11

Chapter 12 Introduction to Three Dimensional Geometry Exercise 12.1, 12.2, 12.3, miscellaneous Solutions

Exercise 12.1 : Solutions of Questions on Page Number : **271**

Q1 :

A point is on the *x*-axis. What are its *y*-coordinates and *z*-coordinates?

Answer :

If a point is on the *x*-axis, then its *y*-coordinates and *z*-coordinates are zero.

Q2 :

A point is in the XZ-plane. What can you say about its *y*-coordinate?

Answer :

If a point is in the XZ plane, then its *y*-coordinate is zero.

Q3 :

Name the octants in which the following points lie:

(1, 2, 3), (4, -2, 3), (4, -2, -5), (4, 2, -5), (-4, 2, -5), (-4, 2, 5),

(-3, -1, 6), (2, -4, -7)

Answer :

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, -2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, -2, -5) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, 2, -5) are positive, positive, and negative respectively. Therefore, this point lies in octant **V**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (-4, 2, -5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (-4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant **II**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (-3, -1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant **III**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (2, -4, -7) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

Q4 :

Fill in the blanks:

Answer :

(i) The *x*-axis and *y*-axis taken together determine a plane known as $\underline{\text{XY} - \text{plane}}$.

(ii) The coordinates of points in the XY-plane are of the form $\underline{(x, y, 0)}$.

(iii) Coordinate planes divide the space into $\underline{\text{eight}}$ octants.

Exercise 12.2 : Solutions of Questions on Page Number : **273**

Q1 :

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (-3, 7, 2) and (2, 4, -1)

(iii) (-1, 3, -4) and (1, -3, 4) (iv) (2, -1, 3) and (-2, 1, 3)

Answer :

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by $\text{PQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$
$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

(ii) Distance between points (â€“3, 7, 2) and (2, 4, â€“1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

(iii) Distance between points (â€“1, 3, â€“4) and (1, â€“3, 4)

$$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$
$$= \sqrt{(2)^2 + (-6)^3 + (8)^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$$

(iv) Distance between points (2, â€“1, 3) and (â€“2, 1, 3)

$$= \sqrt{(-2-2)^2 + (1+1)^2 + (3-3)^2}$$
$$= \sqrt{(-4)^2 + (2)^2 + (0)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

Q2 :

Show that the points (-2, 3, 5), (1, 2, 3) and (7, 0, -1) are collinear.

Answer :

Let points (â€“2, 3, 5), (1, 2, 3), and (7, 0, â€“1) be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$PQ = \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$$
$$= \sqrt{(3)^2+(-1)^2+(-2)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$$
$$= \sqrt{(6)^2+(-2)^2+(-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

$$PR = \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$$
$$= \sqrt{(9)^2+(-3)^2+(-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Here, PQ + QR $= \sqrt{14}+2\sqrt{14}=3\sqrt{14}$ = PR

Hence, points P(â€“2, 3, 5), Q(1, 2, 3), and R(7, 0, â€“1) are collinear.

Q3 :

Verify the following:

(i) (0, 7, -10), (1, 6, -6) and (4, 9, -6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right angled triangle.

(iii) (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram.

Answer :

(i) Let points (0, 7, â€“10), (1, 6, â€“6), and (4, 9, â€“6) be denoted by A, B, and C respectively.

$$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2}$$
$$= \sqrt{(1)^2 + (-1)^2 + (4)^2}$$
$$= \sqrt{1+1+16}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2}$$
$$= \sqrt{(3)^2 + (3)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CA = \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2}$$
$$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2}$$
$$= \sqrt{16+4+16} = \sqrt{36} = 6$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (â€“1, 6, 6), and (â€“4, 9, 6) be denoted by A, B, and C respectively.

$$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$
$$= \sqrt{(-1)^2 + (-1)^2 + (-4)^2}$$
$$= \sqrt{1+1+16} = \sqrt{18}$$
$$= 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2}$$
$$= \sqrt{(-3)^2 + (3)^2 + (0)^2}$$
$$= \sqrt{9+9} = \sqrt{18}$$
$$= 3\sqrt{2}$$

$$CA = \sqrt{(0+4)^2 + (7-9)^2 + (10-6)^2}$$
$$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$$
$$= \sqrt{16+4+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$\text{Now, } AB^2 + BC^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = 18 + 18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (â€“1, 2, 1), (1, â€“2, 5), (4, â€“7, 8), and (2, â€“3, 4) be denoted by A, B, C, and D respectively.

$$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

$$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Q4 :

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, -1).

Answer :

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, â€“1).

Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$
$$\Rightarrow (x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z+1)^2$$

⇒ x^2 â€“ 2x + 1 + y^2 â€“ 4y + 4 + z^2 â€“ 6z + 9 = x^2 â€“ 6x + 9 + y^2 â€“ 4y + 4 + z^2 + 2z + 1

⇒ â€“2x â€“4y â€“ 6z + 14 = â€“6x â€“ 4y + 2z + 14

⇒ â€“ 2x â€“ 6z + 6x â€“ 2z = 0

⇒ 4x â€“ 8z = 0

⇒ x â€“ 2z = 0

Thus, the required equation is x â€“ 2z = 0.

Q5 :

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (-4, 0, 0) is equal to 10.

Answer :

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (â€“4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$$
$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}=10-\sqrt{(x+4)^2+y^2+z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2+y^2+z^2=100-20\sqrt{(x+4)^2+y^2+z^2}+(x+4)^2+y^2+z^2$$
$$\Rightarrow x^2-8x+16+y^2+z^2=100-20\sqrt{x^2+8x+16+y^2+z^2}+x^2+8x+16+y^2+z^2$$
$$\Rightarrow 20\sqrt{x^2+8x+16+y^2+z^2}=100+16x$$
$$\Rightarrow 5\sqrt{x^2+8x+16+y^2+z^2}=(25+4x)$$

On squaring both sides again, we obtain

25 (x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x

⇒ $25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

⇒ $9x^2 + 25y^2 + 25z^2$ â€“ 225 = 0

Thus, the required equation is $9x^2 + 25y^2 + 25z^2$ â€“ 225 = 0.

Exercise 12.3 : Solutions of Questions on Page Number : **277**

Q1 :

Find the coordinates of the point which divides the line segment joining the points (-2, 3, 5) and (1, -4, 6) in the ratio (i) 2:3 internally, (ii) 2:3 externally.

Answer :

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio *m*: *n* are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right).$$

Let R (*x*, *y*, *z*) be the point that divides the line segment joining points(â€“2, 3, 5) and (1, â€“4, 6) internally in the ratio 2:3

$$x=\frac{2(1)+3(-2)}{2+3}, y=\frac{2(-4)+3(3)}{2+3}, \text{ and } z=\frac{2(6)+3(5)}{2+3}$$
$$\text{i.e., } x=\frac{-4}{5}, y=\frac{1}{5}, \text{ and } z=\frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio *m*: *n* are

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right).$$

Let R (*x*, *y*, *z*) be the point that divides the line segment joining points(â€“2, 3, 5) and (1, â€“4, 6) externally in the ratio 2:3

$$x=\frac{2(1)-3(-2)}{2-3}, y=\frac{2(-4)-3(3)}{2-3}, \text{ and } z=\frac{2(6)-3(5)}{2-3}$$
$$\text{i.e., } x=-8, y=17, \text{ and } z=3$$

Thus, the coordinates of the required point are (â€“8, 17, 3).

Q2 :

Given that P (3, 2, -4), Q (5, 4, -6) and R (9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Answer :

Let point Q (5, 4, â€“6) divide the line segment joining points P (3, 2, â€“4) and R (9, 8, â€“10) in the ratio *k*:1.

Therefore, by section formula,

$$(5,4,-6)=\left(\frac{k(9)+3}{k+1},\frac{k(8)+2}{k+1},\frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1}=5$$
$$\Rightarrow 9k+3=5k+5$$
$$\Rightarrow 4k=2$$
$$\Rightarrow k=\frac{2}{4}=\frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Q3 :

Find the ratio in which the YZ-plane divides the line segment formed by joining the points (-2, 4, 7) and (3, -5, 8).

Answer :

Let the YZ planedivide the line segment joining points (â€“2, 4, 7) and (3, â€“5, 8) in the ratio *k*:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1},\frac{k(-5)+4}{k+1},\frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the *x*-coordinate of any point is zero.

$$\frac{3k-2}{k+1}=0$$
$$\Rightarrow 3k-2=0$$
$$\Rightarrow k=\frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Q4 :

Using section formula, show that the points A (2, â€“3, 4), B (â€“1, 2, 1) and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

Answer :

The given points are A (2, â€“3, 4), B (â€“1, 2, 1), and $C\left(0, \frac{1}{3}, 2\right)$.

Let P be a point that divides AB in the ratio *k*:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1}, \frac{k(2)-3}{k+1}, \frac{k(1)+4}{k+1}\right)$$

Now, we find the value of *k* at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$.

i.e., $C\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

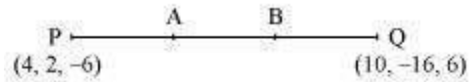
Hence, points A, B, and C are collinear.

Q5 :

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, -6) and Q (10, -16, 6).

Answer :

Let A and B be the points that trisect the line segment joining points P (4, 2, â€“6) and Q (10, â€“16, 6)



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2}, \frac{1(-16)+2(2)}{1+2}, \frac{1(6)+2(-6)}{1+2}\right) = (6, -4, -2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1},\frac{2(-16)+1(2)}{2+1},\frac{2(6)-1(6)}{2+1}\right)=(8,-10,2)$$

Thus, (6, â€“4, â€“2) and (8, â€“10, 2) are the points that trisect the line segment joining points P (4, 2, â€“6) and Q (10, â€“16, 6).

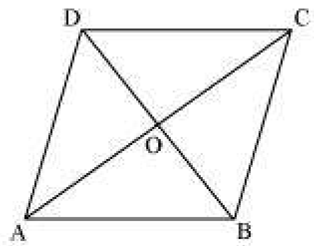
Exercise Miscellaneous : Solutions of Questions on Page Number : **278**

Q1 :

Three vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4) andC (-1, 1, 2). Find the coordinates of the fourth vertex.

Answer :

The three vertices of a parallelogram ABCD are given as A (3, â€“1, 2), B (1, 2, â€“4), and C (â€“1, 1, 2). Let the coordinates of the fourth vertex be D (*x*, *y*, *z*).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴Mid-point of AC = Mid-point of BD

$$\Rightarrow\left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$
$$\Rightarrow(1,0,2)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$
$$\Rightarrow\frac{x+1}{2}=1,\ \frac{y+2}{2}=0,\ \text{and}\ \frac{z-4}{2}=2$$

⇒ *x* = 1, *y* = â€“2, and *z* = 8

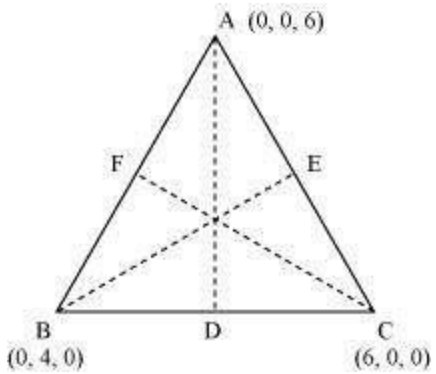
Thus, the coordinates of the fourth vertex are (1, â€“2, 8).

Q2 :

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Answer :

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

∴Coordinates of point D = $\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)$ = (3, 2, 0)

$$AD=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}=\sqrt{9+4+36}=\sqrt{49}=7$$

Since BE is the median, E is the mid-point of AC.

$$\therefore \text{Coordinates of point E}=\left(\frac{0+6}{2},\frac{0+0}{2},\frac{6+0}{2}\right)=(3,0,3)$$

$$BE=\sqrt{(3-0)^2+(0-4)^2+(3-0)^2}=\sqrt{9+16+9}=\sqrt{34}$$

Since CF is the median, F is the mid-point of AB.

$$\therefore \text{Coordinates of point F}=\left(\frac{0+0}{2},\frac{0+4}{2},\frac{6+0}{2}\right)=(0,2,3)$$

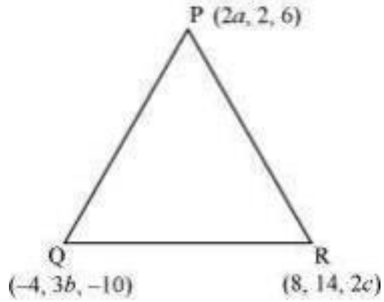
$$\text{Length of CF}=\sqrt{(6-0)^2+(0-2)^2+(0-3)^2}=\sqrt{36+4+9}=\sqrt{49}=7$$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}, \text{ and } 7$.

Q3 :

If the origin is the centroid of the triangle PQR with vertices P (2*a*, 2, 6), Q (-4, 3*b*, -10) and R (8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Answer :



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR $=\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$

It is given that origin is the centroid of ΔPQR.

$$\therefore (0,0,0)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

$$\Rightarrow \frac{2a+4}{3}=0, \frac{3b+16}{3}=0 \text{ and } \frac{2c-4}{3}=0$$

$$\Rightarrow a=-2, b=-\frac{16}{3} \text{ and } c=2$$

Thus, the respective values of *a*, *b*, and *c* are $-2, -\frac{16}{3}, \text{ and } 2.$

Q4 :

Find the coordinates of a point on *y*-axis which are at a distance of $5\sqrt{2}$ from the point P (3, â€“2, 5).

Answer :

If a point is on the *y*-axis, then *x*-coordinate and the *z*-coordinate of the point are zero.

Let A (0, *b*, 0) be the point on the *y*-axis at a distance of $5\sqrt{2}$ from point P (3, â€“2, 5). Accordingly, $AP=5\sqrt{2}$

$$\therefore AP^2 = 50$$
$$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$
$$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$$
$$\Rightarrow b^2 + 4b - 12 = 0$$
$$\Rightarrow b^2 + 6b - 2b - 12 = 0$$
$$\Rightarrow (b+6)(b-2) = 0$$
$$\Rightarrow b = -6 \text{ or } 2$$

Thus, the coordinates of the required points are (0, 2, 0) and (0, â€“6, 0).

Q5 :

A point R with *x*-coordinate 4 lies on the line segment joining the pointsP (2, â€“3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[Hint suppose R divides PQ in the ratio *k*: 1. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$**]**

Answer :

The coordinates of points P and Q are given as P (2, â€“3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio *k*:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the *x*-coordinate of point R is 4.

$$\therefore \frac{8k+2}{k+1} = 4$$
$$\Rightarrow 8k + 2 = 4k + 4$$
$$\Rightarrow 4k = 2$$
$$\Rightarrow k = \frac{1}{2}$$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$

Q6 :

If A and B be the points (3, 4, 5) and (-1, 3, -7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where *k* is a constant.

Answer :

The coordinates of points A and B are given as (3, 4, 5) and (â€“1, 3, â€“7) respectively.

Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$\begin{aligned} PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\ &= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\ &= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50 \end{aligned}$$

$$\begin{aligned} PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\ &= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59 \end{aligned}$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\left(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50\right) + \left(x^2 + 2x + y^2 - 6y + z^2 + 14z + 59\right) = k^2$$
$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$
$$\Rightarrow 2\left(x^2 + y^2 + z^2 - 2x - 7y + 2z\right) = k^2 - 109$$
$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$.