## NCERT Solutions for Class 10 Maths Unit 11

## Constructions Class 10

Unit 11 Constructions Exercise 11.1, 11.2 Solutions

Exercise 11.1 : Solutions of Questions on Page Number : 219
Q1 :

Draw a line segment of length 7.6 cm and divide it in the ratio $5: 8$. Measure the two parts. Give the justification of the construction.

## Answer :

A line segment of length 7.6 cm can be divided in the ratio of $5: 8$ as follows.
Step 1 Draw line segment $A B$ of 7.6 cm and draw a ray $A X$ making an acute angle with line segment $A B$.
Step 2 Locate $13(=5+8)$ points, $A_{1}, A_{2}, A_{3}, A_{4} \ldots \ldots . A_{13}$, on $A X$ such that $A A_{1}=A_{1} A_{2}=A_{2} A_{3}$ and so on.
Step 3 Join $\mathrm{BA}_{13}$.
Step 4 Through the point $A_{5}$, draw a line parallel to $B A_{13}$ (by making an angle equal to $\angle A A_{13} B$ ) at $A_{5}$ intersecting $A B$ at point $C$.
$C$ is the point dividing line segment $A B$ of 7.6 cm in the required ratio of $5: 8$.
The lengths of $A C$ and $C B$ can be measured. It comes out to 2.9 cm and 4.7 cm respectively.


## Justification

The construction can be justified by proving that
$\frac{\mathrm{AC}}{\mathrm{CB}}=\frac{5}{8}$
By construction, we have $A_{5} C \| A_{13} B$. By applying Basic proportionality theorem for the triangle $A A_{13} B$, we obtain

$$
\begin{equation*}
\frac{\mathrm{AC}}{\mathrm{CB}}=\frac{\mathrm{AA}_{5}}{\mathrm{~A}_{5} \mathrm{~A}_{13}} \tag{1}
\end{equation*}
$$

From the figure, it can be observed that $\mathrm{AA}_{5}$ and $\mathrm{A}_{5} \mathrm{~A}_{13}$ contain 5 and 8 equal divisions of line segments respectively.
$\therefore \frac{\mathrm{AA}_{5}}{\mathrm{~A}_{5} \mathrm{~A}_{13}}=\frac{5}{8}$
On comparing equations (1) and (2), we obtain
$\frac{\mathrm{AC}}{\mathrm{CB}}=\frac{5}{8}$
This justifies the construction.

## Q2 :

Construct a triangle of sides $4 \mathrm{~cm}, 5 \mathrm{~cm}$ and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the
corresponding sides of the first triangle.

## Give the justification of the construction.

## Answer :

## Step 1

Draw a line segment $A B=4 \mathrm{~cm}$. Taking point $A$ as centre, draw an arc of 5 cm radius. Similarly, taking point $B$ as its centre, draw an arc of 6 cm radius. These arcs will intersect each other at point $C$. Now, $A C=5 \mathrm{~cm}$ and $\mathrm{BC}=6 \mathrm{~cm}$ and $\triangle A B C$ is the required triangle.

## Step 2

Draw a ray $A X$ making an acute angle with line $A B$ on the opposite side of vertex $C$.

## Step 3

Locate 3 points $A_{1}, A_{2}, A_{3}$ (as 3 is greater between 2 and 3 ) on line $A X$ such that $A A_{1}=A_{1} A_{2}=A_{2} A_{3}$.

## Step 4

Join $B A_{3}$ and draw a line through $A_{2}$ parallel to $B A_{3}$ to intersect $A B$ at point $B^{\prime}$.

## Step 5

Draw a line through $B^{\prime}$ parallel to the line $B C$ to intersect $A C$ at $C^{\prime}$.
$\triangle A B^{\prime} C^{\prime}$ is the required triangle.


## Justification

The construction can be justified by proving that
$\mathrm{AB}^{\prime}=\frac{2}{3} \mathrm{AB}, \mathrm{B}^{\prime} \mathrm{C}^{\prime}=\frac{2}{3} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{2}{3} \mathrm{AC}$
By construction, we have $\mathrm{B}^{\prime} \mathrm{C}^{\prime}| | \mathrm{BC}$
$\therefore \angle A \mathrm{~B}^{\prime} \mathrm{C}^{\prime}=\angle \mathrm{ABC}$ (Corresponding angles)
In $\triangle A B^{\prime} C^{\prime}$ and $\triangle A B C$,
$\angle \mathrm{AB}^{\prime} \mathrm{C}^{\prime}=\angle \mathrm{ABC}$ (Proved above)
$\angle{\overline{B^{\prime}}{ }^{\prime} C^{\prime}}^{\prime}=\angle \mathrm{BAC}$ (Common)
$\therefore \triangle \mathrm{AB}^{\prime} \mathrm{C}^{\prime}$ Ã $\notin \ddot{E} \dagger \hat{\hat{A}^{1} / 4} \Delta \mathrm{ABC}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}^{\prime}}{\mathrm{AB}}=\frac{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}{\mathrm{BC}}=\frac{\mathrm{AC}^{\prime}}{\mathrm{AC}}$
In $\triangle A_{2} B^{\prime}$ and $\triangle A_{3} B$,
$\angle \mathrm{A}_{2} \mathrm{AB}^{\prime}=\angle \mathrm{A}_{3} \mathrm{AB}$ (Common)
$\angle \mathrm{AA}_{2} \mathrm{~B}^{\prime}=\angle \mathrm{AA}_{3} \mathrm{~B}$ (Corresponding angles)
$\therefore \triangle \mathrm{AA}_{2} \mathrm{~B}^{\prime} \mathrm{A} \not \subset \mathrm{E} \dagger \hat{A}^{1} / 4 \Delta \mathrm{AA}_{3} \mathrm{~B}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}^{\prime}}{\mathrm{AB}}=\frac{\mathrm{AA}_{2}}{\mathrm{AA}_{3}}$
$\Rightarrow \frac{\mathrm{AB}^{\prime}}{\mathrm{AB}}=\frac{2}{3}$
From equations (1) and (2), we obtain
$\frac{\mathrm{AB}^{\prime}}{\mathrm{AB}}=\frac{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}{\mathrm{BC}}=\frac{\mathrm{AC}^{\prime}}{\mathrm{AC}}=\frac{2}{3}$
$\Rightarrow \mathrm{AB}^{\prime}=\frac{2}{3} \mathrm{AB}, \mathrm{B}^{\prime} \mathrm{C}^{\prime}=\frac{2}{3} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{2}{3} \mathrm{AC}$
This justifies the construction.

Q3 :
Construct a triangle with sides $5 \mathrm{~cm}, 6 \mathrm{~cm}$ and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the
corresponding sides of the first triangle.
Give the justification of the construction.

## Answer:

## Step 1

Draw a line segment $A B$ of 5 cm . Taking $A$ and $B$ as centre, draw arcs of 6 cm and 7 cm radius respectively. Let these arcs intersect each other at point $C$. $\triangle A B C$ is the required triangle having length of sides as $5 \mathrm{~cm}, 6 \mathrm{~cm}$, and 7 cm respectively.

## Step 2

Draw a ray $A X$ making acute angle with line $A B$ on the opposite side of vertex $C$.

## Step 3

Locate 7 points, $A_{1}, A_{2}, A_{3}, A_{4} A_{5}, A_{6}, A_{7}$ (as 7 is greater between 5and 7), on line $A X$ such that $A A_{1}=A_{1} A_{2}=A_{2} A_{3}=$ $\mathrm{A}_{3} \mathrm{~A}_{4}=\mathrm{A}_{4} \mathrm{~A}_{5}=\mathrm{A}_{5} \mathrm{~A}_{6}=\mathrm{A}_{6} \mathrm{~A}_{7}$.

## Step 4

Join $B A_{5}$ and draw a line through $A_{7}$ parallel to $B A_{5}$ to intersect extended line segment $A B$ at point $B^{\prime}$.

## Step 5

Draw a line through $B^{\prime}$ parallel to $B C$ intersecting the extended line segment $A C$ at $C^{\prime} . \Delta A B^{\prime} C^{\prime}$ is the required triangle.


## Justification

The construction can be justified by proving that
$\mathrm{AB}^{\prime}=\frac{7}{5} \mathrm{AB}, \mathrm{B}^{\prime} \mathrm{C}^{\prime}=\frac{7}{5} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{7}{5} \mathrm{AC}$
In $\triangle A B C$ and $\triangle A B^{\prime} C^{\prime}$,
$\angle A B C=\angle A B^{\prime} C^{\prime}$ (Corresponding angles)
$\angle B A C=\angle B^{\prime} A C^{\prime}$ (Common)
$\therefore \triangle A B C \tilde{A} \phi \ddot{\mathrm{E}} \dagger \hat{A}^{1} / 14 \Delta \mathrm{~B}^{\prime} \mathrm{C}^{\prime}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{BC}}{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}=\frac{\mathrm{AC}}{\mathrm{AC}^{\prime}}$
In $\triangle \mathrm{AA}_{5} \mathrm{~B}$ and $\triangle \mathrm{AA}_{7} \mathrm{~B}^{\prime}$,
$\angle A_{5} A B=\angle A_{7} A^{\prime}$ (Common)
$\angle \mathrm{AA}_{5} \mathrm{~B}=\angle \mathrm{AA}_{7} \mathrm{~B}^{\prime}$ (Corresponding angles)
$\therefore \Delta \mathrm{AA}_{5} \mathrm{~B} \tilde{\mathrm{~A}} c \mathrm{E} \dagger{ }^{\dagger} \hat{A}^{1 / 4} \Delta \mathrm{AA}_{7} \mathrm{~B}^{\prime}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{AA}_{5}}{\mathrm{AA}_{7}}$
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{5}{7}$
On comparing equations (1) and (2), we obtain
$\frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{BC}}{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}=\frac{\mathrm{AC}}{\mathrm{AC}^{\prime}}=\frac{5}{7}$

$$
\mathrm{AB}^{\prime}=\frac{7}{5} \mathrm{AB}, \mathrm{~B}^{\prime} \mathrm{C}^{\prime}=\frac{7}{5} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{7}{5} \mathrm{AC}
$$

This justifies the construction.

## Q4 :

## Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose side

 $1 \frac{1}{2}$ times the corresponding sides of the isosceles triangle.Give the justification of the construction.

## Answer:

Let us assume that $\triangle A B C$ is an isosceles triangle having $C A$ and $C B$ of equal lengths, base $A B$ of 8 cm , and $A D$ is the altitude of 4 cm .

A $\triangle A B^{\prime} C^{\prime}$ whose sides are $\frac{3}{2}_{\text {times of } \triangle A B C \text { can be drawn as follows. }}$.

## Step 1

Draw a line segment $A B$ of 8 cm . Draw arcs of same radius on both sides of the line segment while taking point $A$ and $B$ as its centre. Let these arcs intersect each other at $O$ and $O^{\prime}$. Join $O O^{\prime}$. Let $O O^{\prime}$ intersect $A B$ at $D$.

## Step 2

Taking $D$ as centre, draw an arc of 4 cm radius which cuts the extended line segment $O O^{\prime}$ at point $C$. An isosceles $\triangle A B C$ is formed, having $C D$ (altitude) as 4 cm and $A B$ (base) as 8 cm .

## Step 3

Draw a ray $A X$ making an acute angle with line segment $A B$ on the opposite side of vertex $C$.

## Step 4

Locate 3 points (as 3 is greater between 3 and 2) $A_{1}, A_{2}$, and $A_{3}$ on $A X$ such that $A A_{1}=A_{1} A_{2}=A_{2} A_{3}$.

## Step 5

Join $B A_{2}$ and draw a line through $A_{3}$ parallel to $B A_{2}$ to intersect extended line segment $A B$ at point $B^{\prime}$.

## Step 6

Draw a line through $B^{\prime}$ parallel to $B C$ intersecting the extended line segment $A C$ at $C^{\prime} . \Delta A B^{\prime} C^{\prime}$ is the required triangle.


## Justification

The construction can be justified by proving that
$\mathrm{AB}^{\prime}=\frac{3}{2} \mathrm{AB}, \mathrm{B}^{\prime} \mathrm{C}^{\prime}=\frac{3}{2} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{3}{2} \mathrm{AC}$
In $\triangle A B C$ and $\triangle A B^{\prime} C^{\prime}$,
$\angle A B C=\angle A B^{\prime} C^{\prime}$ (Corresponding angles)
$\angle B A C=\angle B^{\prime} A C '$ (Common)
$\therefore \triangle \mathrm{ABC} \tilde{A} \phi E ̈ \dagger \hat{A}^{1} / 4 \Delta \mathrm{AB}^{\prime} \mathrm{C}^{\prime}(\mathrm{AA}$ similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{BC}}{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}=\frac{\mathrm{AC}}{\mathrm{AC}^{\prime}}$
In $\triangle A_{2} B$ and $\triangle A A_{3} B^{\prime}$,
$\angle \mathrm{A}_{2} \mathrm{AB}=\angle \mathrm{A}_{3} \mathrm{AB}^{\prime}$ (Common)
$\angle A A_{2} B=\angle A A_{3} B^{\prime}$ (Corresponding angles)

$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{AA}_{2}}{\mathrm{AA}_{3}}$
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{2}{3}$
On comparing equations (1) and (2), we obtain

$$
\begin{aligned}
& \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{BC}}{\mathrm{BC}^{\prime} \mathrm{C}^{\prime \prime}}=\frac{\mathrm{AC}}{\mathrm{AC}^{\prime}}=\frac{2}{3} \\
& \Rightarrow \mathrm{AB}^{\prime}=\frac{3}{2} \mathrm{AB}, \mathrm{~B}^{\prime} \mathrm{C}^{\prime}=\frac{3}{2} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{3}{2} \mathrm{AC}
\end{aligned}
$$

This justifies the construction.

Q5 :
Draw a triangle $A B C$ with side $B C=6 \mathrm{~cm}, A B=5 \mathrm{~cm}$ and $\angle A B C=60^{\circ}$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle $A B C$.

Give the justification of the construction.

## Answer:

A $\triangle A^{\prime} B C^{\prime}$ whose sides are $\frac{\frac{3}{4}}{4}$ of the corresponding sides of $\triangle A B C$ can be drawn as follows.
Step 1
Draw a $\triangle A B C$ with side $B C=6 \mathrm{~cm}, A B=5 \mathrm{~cm}$ and $\angle A B C=60^{\circ}$.

## Step 2

Draw a ray $B X$ making an acute angle with $B C$ on the opposite side of vertex $A$.

## Step 3

Locate 4 points (as 4 is greater in 3 and 4 ), $B_{1}, B_{2}, B_{3}, B_{4}$, on line segment $B X$.

## Step 4

Join $B_{4} C$ and draw a line through $B_{3}$, parallel to $B_{4} C$ intersecting $B C$ at $C^{\prime}$.

## Step 5

Draw a line through $C^{\prime}$ parallel to $A C$ intersecting $A B$ at $A^{\prime} . \Delta A^{\prime} B C^{\prime}$ is the required triangle.


## Justification

The construction can be justified by proving
$\mathrm{A}^{\prime} \mathrm{B}=\frac{3}{4} \mathrm{AB}, \mathrm{BC}^{\prime}=\frac{3}{4} \mathrm{BC}, \mathrm{A}^{\prime} \mathrm{C}^{\prime}=\frac{3}{4} \mathrm{AC}$

In $\triangle A^{\prime} B^{\prime}$ and $\triangle A B C$,
$\angle A^{\prime} C^{\prime} B=\angle A C B$ (Corresponding angles)
$\angle A^{\prime} B^{\prime}=\angle A B C$ (Common)
$\therefore \triangle A^{\prime} \mathrm{BC}^{\prime} \mathrm{A} \not \subset E ̈ \dagger \hat{A} 1 / 4 \Delta \mathrm{ABC}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{A}^{\prime} \mathrm{B}}{\mathrm{AB}}=\frac{\mathrm{BC}^{\prime}}{\mathrm{BC}}=\frac{\mathrm{A}^{\prime} \mathrm{C}^{\prime}}{\mathrm{AC}}$
In $\Delta \mathrm{BB}_{3} \mathrm{C}^{\prime}$ and $\triangle \mathrm{BB}_{4} \mathrm{C}$,
$\angle B_{3} B^{\prime}=\angle B_{4} B C$ (Common)
$\angle \mathrm{BB}_{3} \mathrm{C}^{\prime}=\angle \mathrm{BB}_{4} \mathrm{C}$ (Corresponding angles)
$\therefore \Delta \mathrm{BB}_{3} \mathrm{C}^{\prime} \tilde{\mathrm{A}} \not \subset \mathrm{E} \dagger \hat{\mathrm{A}} 1 / 4 \Delta \mathrm{BB}_{4} \mathrm{C}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{BC}^{\prime \prime}}{\mathrm{BC}}=\frac{\mathrm{BB}_{3}}{\mathrm{BB}_{4}}$
$\Rightarrow \frac{\mathrm{BC}^{\prime \prime}}{\mathrm{BC}}=\frac{3}{4}$
From equations (1) and (2), we obtain
$\frac{\mathrm{A}^{\prime} \mathrm{B}}{\mathrm{AB}}=\frac{\mathrm{BC}^{\prime}}{\mathrm{BC}}=\frac{\mathrm{A}^{\prime} \mathrm{C}^{\prime}}{\mathrm{AC}}=\frac{3}{4}$
$\Rightarrow \mathrm{A}^{\prime} \mathrm{B}=\frac{3}{4} \mathrm{AB}, \mathrm{BC}^{\prime}=\frac{3}{4} \mathrm{BC}, \mathrm{A}^{\prime} \mathrm{C}^{\prime}=\frac{3}{4} \mathrm{AC}$
This justifies the construction.

## Q6:

# Draw a triangle $A B C$ with side $B C=7 \mathrm{~cm}, \angle B=45^{\circ}, \angle A=105^{\circ}$. Then, construct a triangle whose sides are 3 times the corresponding side of $\triangle A B C$. Give the justification of the construction. 

## Answer:

$\angle B=45^{\circ}, \angle \mathrm{A}=105^{\circ}$
Sum of all interior angles in a triangle is $180^{\circ}$.
$\angle A+\angle B+\angle C=180^{\circ}$
$105^{\circ}+45^{\circ}+\angle \mathrm{C}=180^{\circ}$
$\angle \mathrm{C}=180^{\circ}-150^{\circ}$
$\angle \mathrm{C}=30^{\circ}$
The required triangle can be drawn as follows.

Step 1
Draw a $\triangle A B C$ with side $B C=7 \mathrm{~cm}, \angle B=45^{\circ}, \angle C=30^{\circ}$.
Step 2
Draw a ray $B X$ making an acute angle with $B C$ on the opposite side of vertex $A$.

## Step 3

Locate 4 points (as 4 is greater in 4 and 3 ), $B_{1}, B_{2}, B_{3}, B_{4}$, on $B X$.

## Step 4

Join $B_{3} C$. Draw a line through $B_{4}$ parallel to $B_{3} C$ intersecting extended $B C$ at $C^{\prime}$.

## Step 5

Through $C^{\prime}$, draw a line parallel to $A C$ intersecting extended line segment at $C^{\prime} . \Delta A^{\prime} B C^{\prime}$ is the required triangle.


Justification
The construction can be justified by proving that
$\mathrm{A}^{\prime} \mathrm{B}=\frac{4}{3} \mathrm{AB}, \mathrm{BC}^{\prime}=\frac{4}{3} \mathrm{BC}, \mathrm{A}^{\prime} \mathrm{C}^{\prime}=\frac{4}{3} \mathrm{AC}$
In $\triangle A B C$ and $\triangle A^{\prime} B C^{\prime}$,
$\angle A B C=\angle A^{\prime} B^{\prime}$ ' (Common)
$\angle A C B=\angle A^{\prime} C^{\prime} B$ (Corresponding angles)
$\therefore \triangle \mathrm{ABC} \tilde{A} \not \subset E ̈ \dagger \hat{A}^{1} 1 / 4 \Delta \mathrm{~A}^{\prime} \mathrm{BC}^{\prime}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{A}^{\prime} \mathrm{B}}=\frac{\mathrm{BC}}{\mathrm{BC}^{\prime}}=\frac{\mathrm{AC}}{\mathrm{A}^{\prime} \mathrm{C}^{\prime}}$
In $\triangle \mathrm{BB}_{3} \mathrm{C}$ and $\triangle \mathrm{BB}_{4} \mathrm{C}^{\prime}$,
$\angle B_{3} B C=\angle B_{4} B C^{\prime}$ (Common)
$\angle \mathrm{BB}_{3} \mathrm{C}=\angle \mathrm{BB}_{4} \mathrm{C}^{\prime}$ (Corresponding angles)
$\therefore \Delta \mathrm{BB}_{3} \mathrm{C}$ Ã $\not \subset \ddot{\mathrm{E}} \dagger \hat{\mathrm{A}} 1 / 4 \Delta \mathrm{BB}_{4} \mathrm{C}^{\prime}$ (AA similarity criterion)

$$
\begin{align*}
& \Rightarrow \frac{\mathrm{BC}}{\mathrm{BC}^{\prime}}=\frac{\mathrm{BB}_{3}}{\mathrm{BB}_{4}} \\
& \Rightarrow \frac{\mathrm{BC}}{\mathrm{BC}^{\prime}}=\frac{3}{4} \tag{2}
\end{align*}
$$

On comparing equations (1) and (2), we obtain

$$
\begin{aligned}
& \frac{\mathrm{AB}}{\mathrm{~A}^{\prime} \mathrm{B}}=\frac{\mathrm{BC}}{\mathrm{BC}^{\prime}}=\frac{\mathrm{AC}}{\mathrm{~A}^{\prime} \mathrm{C}^{\prime}}=\frac{3}{4} \\
& \Rightarrow \\
& \mathrm{~A}^{\prime} \mathrm{B}=\frac{4}{3} \mathrm{AB}, \mathrm{BC}^{\prime}=\frac{4}{3} \mathrm{BC}, \mathrm{~A}^{\prime} \mathrm{C}^{\prime}=\frac{4}{3} \mathrm{AC}
\end{aligned}
$$

This justifies the construction.

## Q7 :

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm . the construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle. Give the justification of the construction.

## Answer:

It is given that sides other than hypotenuse are of lengths 4 cm and 3 cm . Clearly, these will be perpendicular to each other.

The required triangle can be drawn as follows.

## Step 1

Draw a line segment $A B=4 \mathrm{~cm}$. Draw a ray $S A$ making $90^{\circ}$ with it.

## Step 2

Draw an arc of 3 cm radius while taking $A$ as its centre to intersect $S A$ at $C$. Join $B C$. $\triangle A B C$ is the required triangle.

## Step 3

Draw a ray AX making an acute angle with AB , opposite to vertex C .

## Step 4

Locate 5 points (as 5 is greater in 5 and 3 ), $A_{1}, A_{2}, A_{3}, A_{4}, A_{5}$, on line segment $A X$ such that $A A_{1}=A_{1} A_{2}=A_{2} A_{3}=A_{3} A_{4}=$ $\mathrm{A}_{4} \mathrm{~A}_{5}$.

## Step 5

Join $A_{3} B$. Draw a line through $A_{5}$ parallel to $A_{3} B$ intersecting extended line segment $A B$ at $B^{\prime}$.

## Step 6

Through $B^{\prime}$, draw a line parallel to $B C$ intersecting extended line segment $A C$ at $C^{\prime} . \Delta A B^{\prime} C^{\prime}$ is the required triangle.


## Justification

The construction can be justified by proving that
$\mathrm{AB}^{\prime}=\frac{5}{3} \mathrm{AB}, \mathrm{B}^{\prime} \mathrm{C}^{\prime}=\frac{5}{3} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{5}{3} \mathrm{AC}$
In $\triangle A B C$ and $\triangle A B^{\prime} C^{\prime}$,
$\angle A B C=\angle A B^{\prime} C^{\prime}$ (Corresponding angles)
$\angle B A C=\angle B^{\prime} A C^{\prime}$ (Common)
$\therefore \triangle \mathrm{ABC} \tilde{A} \not \subset E ̈ \dagger \hat{A}^{1} 1 / 4 \Delta \mathrm{AB}^{\prime} \mathrm{C}^{\prime}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{BC}}{\mathrm{B}^{\prime} \mathrm{C}^{\prime}}=\frac{\mathrm{AC}}{\mathrm{AC}^{\prime}}$

In $\triangle A A_{3} B$ and $\triangle A A_{5} B^{\prime}$,
$\angle \mathrm{A}_{3} \mathrm{AB}=\angle \mathrm{A}_{5} \mathrm{AB}^{\prime}$ (Common)
$\angle \mathrm{AA}_{3} \mathrm{~B}=\angle \mathrm{AA}_{5} \mathrm{~B}^{\prime}$ (Corresponding angles)
$\therefore \Delta \mathrm{AA}_{3} \mathrm{~B}$ Ã $¢ E ̈ \dagger \hat{A}^{1} / 4, \Delta \mathrm{AA}_{5} \mathrm{~B}^{\prime}$ (AA similarity criterion)
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{AA}_{3}}{\mathrm{AA}_{5}}$
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{3}{5}$
On comparing equations (1) and (2), we obtain
$\frac{\mathrm{AB}}{\mathrm{AB}^{\prime}}=\frac{\mathrm{BC}}{\mathrm{BC}^{\prime}{ }^{\prime}}=\frac{\mathrm{AC}}{\mathrm{AC}^{\prime}}=\frac{3}{5}$
$\Rightarrow \mathrm{AB}^{\prime}=\frac{5}{3} \mathrm{AB}, \mathrm{B}^{\prime} \mathrm{C}^{\prime}=\frac{5}{3} \mathrm{BC}, \mathrm{AC}^{\prime}=\frac{5}{3} \mathrm{AC}$
Page \| 12 This justifies the construction.

Exercise 11.2: Solutions of Questions on Page Number : 221
Q1 :

Draw a circle of radius 6 cm . From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Give the justification of the construction.

## Answer:

A pair of tangents to the given circle can be constructed as follows.

## Step 1

Taking any point O of the given plane as centre, draw a circle of 6 cm radius. Locate a point $\mathrm{P}, 10 \mathrm{~cm}$ away from O . Join OP.

## Step 2

Bisect OP. Let M be the mid-point of PO.

## Step 3

Taking M as centre and MO as radius, draw a circle.

## Step 4

Let this circle intersect the previous circle at point $Q$ and $R$.

## Step 5

Join $P Q$ and $P R$. $P Q$ and $P R$ are the required tangents.


The lengths of tangents PQ and PR are 8 cm each.

## Justification

The construction can be justified by proving that $P Q$ and $P R$ are the tangents to the circle (whose centre is $O$ and radius is 6 cm ). For this, join OQ and OR.

$\angle \mathrm{PQO}$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.
$\therefore \angle \mathrm{PQO}=90^{\circ}$
$\Rightarrow \mathrm{OQ} \perp \mathrm{PQ}$
Since $O Q$ is the radius of the circle, $P Q$ has to be a tangent of the circle. Similarly, $P R$ is a tangent of the circle.

## Q2 :

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. Give the justification of the construction.

## Answer :

Tangents on the given circle can be drawn as follows.

## Step 1

Draw a circle of 4 cm radius with centre as O on the given plane.

## Step 2

Draw a circle of 6 cm radius taking O as its centre. Locate a point P on this circle and join OP .

## Step 3

Bisect OP. Let M be the mid-point of PO.

## Step 4

Taking M as its centre and MO as its radius, draw a circle. Let it intersect the given circle at the points Q and R .

## Step 5

Join PQ and PR. PQ and PR are the required tangents.


It can be observed that PQ and PR are of length 4.47 cm each.
In $\triangle$ PQO,
Since $P Q$ is a tangent,
$\angle \mathrm{PQO}=90^{\circ}$
$\mathrm{PO}=6 \mathrm{~cm}$
$\mathrm{QO}=4 \mathrm{~cm}$
Applying Pythagoras theorem in $\triangle \mathrm{PQO}$, we obtain

```
PQ +}+\mp@subsup{QO}{}{2}=P\mp@subsup{PQ}{}{2
PQ 2}+(4\mp@subsup{)}{}{2}=(6\mp@subsup{)}{}{2
PQ 2}+16=3
PQ 2}=36-1
PQ }=2
PQ =2\sqrt{}{5}
PQ = 4.47 cm
```

Justification

The construction can be justified by proving that $P Q$ and $P R$ are the tangents to the circle (whose centre is $O$ and radius is 4 cm ). For this, let us join $O Q$ and $O R$.

$\angle P Q O$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.
$\therefore \angle \mathrm{PQO}=90^{\circ}$

Since $O Q$ is the radius of the circle, $P Q$ has to be a tangent of the circle. Similarly, $P R$ is a tangent of the circle.

## Q3 :

Draw a circle of radius 3 cm . Take two points $P$ and $Q$ on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points $P$ and $Q$. Give the justification of the construction.

## Answer :

The tangent can be constructed on the given circle as follows.

## Step 1

Taking any point $O$ on the given plane as centre, draw a circle of 3 cm radius.

## Step 2

Take one of its diameters, PQ , and extend it on both sides. Locate two points on this diameter such that $\mathrm{OR}=\mathrm{OS}=7$ cm

## Step 3

Bisect OR and OS. Let T and U be the mid-points of OR and OS respectively.

## Step 4

Taking $T$ and $U$ as its centre and with TO and $U O$ as radius, draw two circles. These two circles will intersect the circle at point V, W, X, Y respectively. Join RV, RW, SX, and SY. These are the required tangents.


## Justification

The construction can be justified by proving that RV, RW, SY, and SX are the tangents to the circle (whose centre is O and radius is 3 cm ). For this, join OV, OW, OX, and OY.

$\angle R V O$ is an angle in the semi-circle. We know that angle in a semi-circle is a right angle.
$\therefore \angle \mathrm{RVO}=90^{\circ}$
$\Rightarrow \mathrm{OV} \perp \mathrm{RV}$
Since OV is the radius of the circle, RV has to be a tangent to the circle.
Similarly, it can be shown that RW, SX, and SY are the tangents of the circle.

## Q4 :

Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of $60^{\circ}$. Give the justification of the construction.

## Answer:

The tangents can be constructed in the following manner:

## Step 1

Draw a circle of radius 5 cm and with centre as O .

## Step 2

Take a point $A$ on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

## Step 3

Draw a radius OB , making an angle of $120^{\circ}\left(180^{\circ}-60^{\circ}\right)$ with OA .

## Step 4

Draw a perpendicular to $O B$ at point $B$. Let both the perpendiculars intersect at point $P$. $P A$ and $P B$ are the required tangents at an angle of $60^{\circ}$.


## Justification

The construction can be justified by proving that $\angle \mathrm{APB}=60^{\circ}$
By our construction
$\angle \mathrm{OAP}=90^{\circ}$
$\angle \mathrm{OBP}=90^{\circ}$
And $\angle \mathrm{AOB}=120^{\circ}$
We know that the sum of all interior angles of a quadrilateral $=360^{\circ}$
$\angle \mathrm{OAP}+\angle \mathrm{AOB}+\angle \mathrm{OBP}+\angle \mathrm{APB}=360^{\circ}$
$90^{\circ}+120^{\circ}+90^{\circ}+\angle \mathrm{APB}=360^{\circ}$
$\angle \mathrm{APB}=60^{\circ}$
This justifies the construction.

## Q5:

Draw a line segment $A B$ of length 8 cm . Taking $A$ as centre, draw a circle of radius 4 cm and taking $B$ as centre, draw another circle of radius 3 cm . Construct tangents to each circle from the centre of the other circle. Give the justification of the construction.

## Answer:

The tangents can be constructed on the given circles as follows.

## Step 1

Draw a line segment $A B$ of 8 cm . Taking $A$ and $B$ as centre, draw two circles of 4 cm and 3 cm radius.

## Step 2

Bisect the line $A B$. Let the mid-point of $A B$ be $C$. Taking $C$ as centre, draw a circle of $A C$ radius which will intersect the circles at points $P, Q, R$, and $S$. Join $B P, B Q, A S$, and $A R$. These are the required tangents.


## Justification

The construction can be justified by proving that AS and AR are the tangents of the circle (whose centre is B and radius is 3 cm ) and BP and BQ are the tangents of the circle (whose centre is $A$ and radius is 4 cm ). For this, join $A P$, $A Q, B S$, and $B R$.

$\angle$ ASB is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.
$\therefore \angle \mathrm{ASB}=90^{\circ}$
$\Rightarrow B S \perp A S$
Since $B S$ is the radius of the circle, $A S$ has to be a tangent of the circle. Similarly, $A R, B P$, and $B Q$ are the tangents.

## Q6 :

Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circles. Give the justification of the construction.

## Answer:

The required tangents can be constructed on the given circle as follows.

## Step 1

Draw a circle with the help of a bangle.

## Step 2

Take a point $P$ outside this circle and take two chords QR and ST.
Step 3
Draw perpendicular bisectors of these chords. Let them intersect each other at point O .

Join PO and bisect it. Let $U$ be the mid-point of PO. Taking $U$ as centre, draw a circle of radius OU, which will intersect the circle at V and W. Join PV and PW.

PV and PW are the required tangents.


## Justification

The construction can be justified by proving that PV and PW are the tangents to the circle. For this, first of all, it has to be proved that O is the centre of the circle. Let us join OV and OW .


We know that perpendicular bisector of a chord passes through the centre. Therefore, the perpendicular bisector of chords QR and ST pass through the centre. It is clear that the intersection point of these perpendicular bisectors is the centre of the circle. $\angle \mathrm{PVO}$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.
$\therefore \angle \mathrm{PVO}=90^{\circ}$
$\Rightarrow \mathrm{OV} \perp \mathrm{PV}$
Since OV is the radius of the circle, PV has to be a tangent of the circle. Similarly, PW is a tangent of the circle.

## Q7 :

Let $A B C$ be a right triangle in which $A B=6 \mathrm{~cm}, B C=8 \mathrm{~cm}$ and $\angle B=90^{\circ}$. $B D$ is the perpendicular from $B$ on $A C$. The circle through $B, C$, and $D$ is drawn. Construct the tangents from $A$ to this circle. Give the justification of the construction.

## Answer:

Consider the following situation. If a circle is drawn through $B, D$, and $C, B C$ will be its diameter as $\angle B D C$ is of measure $90^{\circ}$. The centre $E$ of this circle will be the mid-point of $B C$.


The required tangents can be constructed on the given circle as follows.

## Step 1

Join AE and bisect it. Let $F$ be the mid-point of $A E$.

## Step 2

Taking $F$ as centre and $F E$ as its radius, draw a circle which will intersect the circle at point $B$ and $G$. Join AG. $A B$ and $A G$ are the required tangents.


## Justification

The construction can be justified by proving that $A G$ and $A B$ are the tangents to the circle. For this, join EG.

$\angle A G E$ is an angle in the semi-circle. We know that an angle in a semi-circle is a right angle.
$\therefore \angle \mathrm{AGE}=90^{\circ}$
$\Rightarrow \mathrm{EG} \perp \mathrm{AG}$
Since $E G$ is the radius of the circle, $A G$ has to be a tangent of the circle.

Already, $\angle \mathrm{B}=90^{\circ}$
$\Rightarrow \mathrm{AB} \perp \mathrm{BE}$
Since $B E$ is the radius of the circle, $A B$ has to be a tangent of the circle.

